

# Nonextensive statistical effects in the hadron to quark-gluon phase transition

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We investigate the relativistic equation of state of hadronic matter and quark-gluon plasma at finite temperature and baryon density in the framework of the nonextensive statistical mechanics, characterized by power-law quantum distributions. We study the phase transition from hadronic matter to quark-gluon plasma by requiring the Gibbs conditions on the global conservation of baryon number and electric charge fraction. We show that nonextensive statistical effects play a crucial role in the equation of state and in the formation of mixed phase also for small deviations from the standard Boltzmann-Gibbs statistics.

## I. INTRODUCTION

The physics of high energy heavy ion collisions is a goldmine of problems in statistical mechanics and thermodynamics due to a large average number of particles involved and possible phase transition phenomena in the hot and dense fireball created during the collisions [1]. In relativistic heavy ion collisions the baryon density can reach values of a few times the saturation nuclear density and/or high temperatures. Furthermore, the future CBM (Compressed Baryonic Matter) experiment of FAIR (Facility of Antiproton and Ion Research) at GSI Darmstadt, will make possible to create compressed baryonic matter with a high net baryon density [2–4]. In this direction interesting results have been obtained at low SPS energy and are foreseen at a low-energy scan at RHIC [5–9].

Lattice calculations predict a critical phase transition from hadronic matter to quark-gluon plasma (QGP) at temperature  $T_c$  of about 170 MeV, corresponding to a critical energy density  $\epsilon_c \approx 1 \text{ GeV/fm}^3$  [10]. On the other hand, in dense nuclear matter, baryons are forced to stay so close one to another that they would overlap. At large densities, constituent quarks are shared by neighboring baryons and should eventually become mobile over a distance larger than the typical size of one baryon. This means that quarks become deconfined and that at large densities and/or high temperatures they are the real degrees of freedom of strongly interacting matter instead of baryons. The process of deconfinement and the EOS of quark-gluon matter can in principle be described by quantum chromodynamics. However, in energy density range reached in relativistic heavy-ion collisions, non-perturbative effects in the complex theory of QCD are not negligible [10]. The generated QGP in the early stages of the collisions does not at all resemble a quasi-ideal gas of quarks and gluons because strongly dynamical correlations are present, including long-range interactions [1, 11–14]. In the absence of a converging method to approach QCD at finite density one often turns to (effective) model investigations. Various results from QCD inspired models indicate that, increasing the baryon chemical potential in the phase diagram, a region of non-singular but rapid cross-over of thermodynamic observable around a quasi-critical temperature, leads to a critical endpoint (CEP), beyond which the system shows a first order phase transition from confined to deconfined matter. The existence or exclusion of a CEP has not yet been confirmed by QCD lattice simulations. Actually, there are some extrapolation techniques to finite chemical potentials [15], although the precise location of the CEP is still a matter of debate [16]. For example, in Ref. [17], the authors estimate the values  $T^{\text{CEP}} = 162 \text{ MeV}$  and  $\mu^{\text{CEP}} = 360 \text{ MeV}$ . Such a CEP can be in principle detected in future high-energy compressed nuclear matter experiments.

Recently, there is an increasing evidence that the nonextensive statistical mechanics, proposed by Tsallis, can be considered as an appropriate basis to deal with physical phenomena where strong dynamical correlations, long-range interactions and microscopic memory effects take place [18–21]. A considerable variety of physical applications involve a quantitative agreement between experimental data and theoretical models based on Tsallis thermostatics. In particular, in the last years there is a growing interest to high energy physics applications of nonextensive statistics [22] and several authors have outlined the possibility that experimental observations in relativistic heavy ion collisions can reflect nonextensive statistical behaviors [23–35].

The existence of nonextensive statistical effects should strongly affects the finite temperature and density nuclear equation of state (EOS) [36–38]. In fact, by varying temperature and density, the EOS reflects in terms of the macroscopic thermodynamical variables the microscopic interactions of the different phases of nuclear matter. The extraction of information about the EOS at different densities and temperatures by means of heavy ion collisions is a very difficult task and can be realized only indirectly by comparing the experimental data with different theoretical models, such as, for example, fluid-dynamical models [39]. Related to this aspect, it is relevant to observe that a relativistic kinetic nonextensive theory [40] and a nonextensive version of a hydrodynamic model for multiparticle production processes have been proposed [41]. Very recently, nonextensive statistical effects on the hadronic EOS have been investigated by means of a Walecka type relativistic mean field model [42]. Furthermore, a nonextensive

version of Nambu-Jona-Lasinio model [43] and the effects on color superconducting phase for two quark flavors due to a change to Tsallis statistics have been studied [44].

The main goal of this paper is to study how nonextensive statistical effects influence, from a phenomenological point of view, the nuclear EOS and, as a consequence, the relative phase transition at finite temperature and density reachable in high-energy heavy-ion collisions. Focusing our investigation to lower temperatures and higher baryon chemical potentials than the corresponding CEP values, a mixed phase of hadrons, quarks and gluons can be formed following the Gibbs conditions for the phase equilibrium. If, in general, we consider a substance composed of two conserved "charges", like the baryon number and the isospin charge in heavy ion collisions at finite baryon density, the ratio between the two charges is fixed only as long as the system remains in one of the two pure phases. In the mixed phase, the concentration in each of the regions of one phase or the other may be different. Their values are restricted only by the conservation on the total charge numbers. The essential point is that conservation laws in chemical thermodynamics are global, not local. The main result of this formalism is that, different from the so-called Maxwell construction, the pressure in the mixed phase is not constant and therefore the nuclear incompressibility, for example, does not vanish [45].

Furthermore, the scenario we are going to explore corresponds to the situation realized in heavy ion collisions experiments at not too high energy where finite temperature and high compressed baryon density is reached. In this condition, a not large fraction of strangeness can be produced and, therefore, we will limit ourselves to study the deconfinement transition from hadronic matter into up and down quark matter [46–48]. We aspect that, in the range of temperature and density considered, the presence of strange particles does not significantly affect the main conclusions regarding the relevance of nonextensive statistical effects on the nuclear EOS.

The paper is organized as follows. In Section II, we introduce the basic formalism of the nonextensive statistics. In Section III, we study the nonextensive hadronic EOS for symmetric and asymmetric nuclear matter and we explore the behavior of meson fields in presence of small deviations from the standard statistics. In Section IV, we investigate nonextensive proprieties of the quark-gluon EOS. In Section V, we study the hadron to quark-gluon phase transition and the consequent formation of a mixed phase, mainly focusing our study in the variation of the first critical transition density for various set of parameters. Finally, we summarize our conclusions in Section VI.

## II. BASIC ASSUMPTIONS IN NONEXTENSIVE STATISTICS

Nonextensive statistical mechanics introduced by Tsallis is a generalization of the common Boltzmann-Gibbs statistical mechanics [18–20]. It is based upon the introduction of the following entropy

$$S_q[f] = \frac{1}{q-1} \left( 1 - \int [f(\mathbf{x})]^q d\Omega \right), \quad \left( \int f(\mathbf{x}) d\Omega = 1 \right), \quad (1)$$

where  $f(\mathbf{x})$  stands for a normalized probability distribution,  $\mathbf{x}$  and  $d\Omega$  denoting, respectively, a generic point and the volume element in the corresponding phase space. Here and in the following we set the Boltzmann and the Planck constant equal to unity. The real parameter  $q$  determines the degree of non-additivity exhibited by the entropy form (1).

The generalized entropy has the usual properties of positivity, equiprobability, concavity and irreversibility, preserves the whole mathematical structure of thermodynamics (Legendre transformations). In the limit  $q \rightarrow 1$ , the entropic form (1) becomes additive and reduces to the standard Boltzmann-Gibbs entropy

$$S_1 = - \int f(\mathbf{x}) \ln f(\mathbf{x}) d\Omega. \quad (2)$$

Peculiarity of the Tsallis generalized thermostatics is that if we have two statistically independent subsystems  $A$  e  $B$ , described, respectively, by the individual probability density  $f^{(A)}$  and  $f^{(B)}$  and we call  $f^{(A+B)}(\mathbf{x}_A, \mathbf{x}_B) = f^{(A)}(\mathbf{x}_A) f^{(B)}(\mathbf{x}_B)$  the joint probability density of a composite system  $A+B$ , the nonadditive (nonextensive) character of  $S_q$  is summarized in the relation [20]

$$S_q[f^{(A+B)}] = S_q[f^{(A)}] + S_q[f^{(B)}] + (1-q)S_q[f^{(A)}] S_q[f^{(B)}]. \quad (3)$$

In the limit  $q \rightarrow 1$ , the third term in right hand side of Eq.(3) vanishes and the above equation reduces to the well-known additivity (extensivity) relation of the Boltzmann-Gibbs logarithmic entropy. Here, the word nonextensive should be associated with the fact that the total energy of long-range-interacting mechanical systems is nonextensive, in contrast with the case of short-range-interacting systems, whose total energy is extensive in the thermodynamical sense [20].

Second crucial assumption on nonextensive statistics is the introduction of the  $q$ -mean value (or escort mean value) of a physical observable  $A(\mathbf{x})$

$$\langle A \rangle_q = \frac{\int A(\mathbf{x}) [f(\mathbf{x})]^q d\Omega}{\int [f(\mathbf{x})]^q d\Omega}. \quad (4)$$

The probability distribution can be obtained maximizing the measure  $S_q$  under appropriate constraints related to the previous definition of the  $q$ -mean value. In this context, it is important to observe that the Tsallis classical distribution can be seen as a superposition of Boltzmann distributions with different temperatures which have a mean value corresponding to the temperature appearing in the Tsallis distribution. The nonextensive  $q$  parameter is related to the fluctuation in the temperature and describes the spread around the average value of the Boltzmann temperature [27].

Following the above prescriptions, it is possible to obtain the associate quantum mean occupation number of particles species  $i$  in a grand canonical ensemble. For a dilute gas of particles and for small deviations from the standard statistics ( $q \approx 1$ ), it can be written as [49, 50]

$$n_i = \frac{1}{[1 + (q-1)\beta(E_i - \mu_i)]^{1/(q-1)} \pm 1}, \quad (5)$$

where  $\beta = 1/T$  and the sign (+1) is for fermions and (-1) for bosons. Naturally, for  $q \rightarrow 1$  the above quantum distribution reduces to the standard Fermi-Dirac and Bose-Einstein distribution. Let us observe that nonextensive statistical effects vanishes approaching to zero temperature. This is the reason for which nonextensive effects could be significantly relevant in high energy heavy ion collision and probably, in the protoneutron star. In addition, in high density quark-gluon matter the color magnetic field remains unscreened (in leading order) and long-range color magnetic interaction should be present at any finite temperature, thus QGP appears to be an ideal candidate for finding some nonextensive behavior.

Finally, let us observe that when the entropic  $q$  parameter is smaller than one, the above distribution have a natural high energy cut-off which implies that the energy tail is depleted; when  $q$  is greater than one, the cut-off is absent and the energy tail of the particle distribution (for fermions and bosons) is enhanced. Hence the nonextensive statistics entails a sensible difference of the power-law particle distribution shape in the high energy region with respect to the standard statistics. In this context, it is relevant to observe that in Ref. [43], the authors postulate a modified quantum distribution function for fermions and bosons at the scope of satisfy the particle-hole symmetry, both for  $q > 1$  and  $q < 1$ . In the present work we will focus our study for small deviations from the standard statistics and for values of  $q > 1$ , because these values were obtained in several phenomenological studies in high energy heavy ion collisions (see, for example, Ref.s [24, 32–35]). We have explicitly verified that, for the values  $q > 1$  considered in this investigation, the prescription introduced in Ref. [43] does not affect the results that we are going to obtain and, therefore, we adopt the original formulation of nonextensive statistics. Furthermore, it is proper to remember that in a relativistic mean field theory, considered in this investigation, baryons are assumed as Dirac quasiparticles moving in classical meson fields, the field operator are replaced by their expectation values and the contributions coming from the Dirac sea are neglected.

### III. NONEXTENSIVE HADRONIC EQUATION OF STATE

In this Section we study the nonextensive hadronic EOS in the framework of a relativistic mean field theory in which nucleons interact through the nuclear force mediated by the exchange of virtual isoscalar-scalar ( $\sigma$ ), isoscalar-vector ( $\omega$ ) and isovector-vector ( $\rho$ ) meson fields [51–53]. As quoted in the Introduction, a similar approach has been studied in Ref. [42] for pure neutron matter and for symmetric nuclear matter (thus, without considering the effects of the  $\rho$  meson field) focusing principally the attention to a different range of density and temperature considered in this paper. Here, we are going to study the hadronic EOS at the scope of emphasize several features previously not investigated that result to be crucial for our following purposes.

The Lagrangian density describing hadronic matter can be written as

$$\mathcal{L} = \mathcal{L}_{QHD} + \mathcal{L}_{\text{qfm}}, \quad (6)$$

where [53]

$$\begin{aligned} \mathcal{L}_{QHD} = & \bar{\psi}[i\gamma_\mu\partial^\mu - (M - g_\sigma\sigma) - g_\omega\gamma_\mu\omega^\mu - g_\rho\gamma^\mu\vec{\tau} \cdot \vec{\rho}_\mu]\psi + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) \\ & - U(\sigma) + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{2}m_\rho^2\vec{\rho}_\mu \cdot \vec{\rho}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\vec{G}_{\mu\nu}\vec{G}^{\mu\nu}, \end{aligned} \quad (7)$$

and  $M = 939$  MeV is the vacuum baryon mass. The field strength tensors for the vector mesons are given by the usual expressions  $F_{\mu\nu} \equiv \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ ,  $\vec{G}_{\mu\nu} \equiv \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu$ , and  $U(\sigma)$  is a nonlinear potential of  $\sigma$  meson

$$U(\sigma) = \frac{1}{3}a\sigma^3 + \frac{1}{4}b\sigma^4, \quad (8)$$

usually introduced to achieve a reasonable compression modulus for equilibrium nuclear matter.

Following Ref.s [54, 55],  $\mathcal{L}_{\text{qfm}}$  in Eq.(6) is related to a (quasi) free gas of pions with an effective chemical potential (see below for details).

The field equations in a mean field approximation are

$$(i\gamma_\mu \partial^\mu - (M - g_\sigma \sigma) - g_\omega \gamma^0 \omega - g_\rho \gamma^0 \tau_3 \rho) \psi = 0, \quad (9)$$

$$m_\sigma^2 \sigma + a\sigma^2 + b\sigma^3 = g_\sigma \langle \bar{\psi} \psi \rangle = g_\sigma \rho_S, \quad (10)$$

$$m_\omega^2 \omega = g_\omega \langle \bar{\psi} \gamma^0 \psi \rangle = g_\omega \rho_B, \quad (11)$$

$$m_\rho^2 \rho = g_\rho \langle \bar{\psi} \gamma^0 \tau_3 \psi \rangle = g_\rho \rho_I, \quad (12)$$

where  $\sigma = \langle \sigma \rangle$ ,  $\omega = \langle \omega^0 \rangle$  and  $\rho = \langle \rho_3^0 \rangle$  are the nonvanishing expectation values of meson fields,  $\rho_I$  is the total isospin density,  $\rho_B$  and  $\rho_S$  are the baryon density and the baryon scalar density, respectively. They are given by

$$\rho_B = 2 \sum_{i=n,p} \int \frac{d^3k}{(2\pi)^3} [n_i(k) - \bar{n}_i(k)], \quad (13)$$

$$\rho_S = 2 \sum_{i=n,p} \int \frac{d^3k}{(2\pi)^3} \frac{M_i^*}{E_i^*} [n_i^q(k) + \bar{n}_i^q(k)], \quad (14)$$

where  $n_i(k)$  and  $\bar{n}_i(k)$  are the  $q$ -deformed fermion particle and antiparticle distributions given in Eq.(5); more explicitly, in this context, we have

$$n_i(k) = \frac{1}{[1 + (q-1)\beta(E_i^*(k) - \mu_i^*)]^{1/(q-1)} + 1}, \quad (15)$$

$$\bar{n}_i(k) = \frac{1}{[1 + (q-1)\beta(E_i^*(k) + \mu_i^*)]^{1/(q-1)} + 1}. \quad (16)$$

The nucleon effective energy is defined as  $E_i^*(k) = \sqrt{k^2 + M_i^{*2}}$ , where  $M_i^* = M_i - g_\sigma \sigma$ . The effective chemical potentials  $\mu_i^*$  are given in terms of the meson fields as follows

$$\mu_i^* = \mu_i - g_\omega \omega - \tau_{3i} g_\rho \rho, \quad (17)$$

where  $\mu_i$  are the thermodynamical chemical potentials  $\mu_i = \partial \epsilon / \partial \rho_i$ . At zero temperature they reduce to the Fermi energies  $E_{Fi} \equiv \sqrt{k_{Fi}^2 + M_i^{*2}}$  and the nonextensive statistical effects disappear. The meson fields are obtained as a solution of the field equations in mean field approximation and the related meson-nucleon couplings ( $g_\sigma$ ,  $g_\omega$  and  $g_\rho$ ) are the free parameters of the model. In the following, they will be fixed to the parameters set marked as GM2 of Ref.[53].

The thermodynamical quantities can be obtained from the thermodynamic potential in the standard way. More explicitly, the baryon pressure  $P_B$  and the energy density  $\epsilon_B$  can be written as

$$P_B = \frac{2}{3} \sum_{i=n,p} \int \frac{d^3k}{(2\pi)^3} \frac{k^2}{E_i^*(k)} [n_i^q(k) + \bar{n}_i^q(k)] - \frac{1}{2} m_\sigma^2 \sigma^2 - U(\sigma) + \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{2} m_\rho^2 \rho^2, \quad (18)$$

$$\epsilon_B = 2 \sum_{i=n,p} \int \frac{d^3k}{(2\pi)^3} E_i^*(k) [n_i^q(k) + \bar{n}_i^q(k)] + \frac{1}{2} m_\sigma^2 \sigma^2 + U(\sigma) + \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{2} m_\rho^2 \rho^2. \quad (19)$$

It is important to observe that Eq.s(14), (18) and (19) apply to  $n_i^q \equiv (n_i)^q$  rather than  $n_i$  itself, this is a direct consequence of the basic prescription related to the  $q$ -mean expectation value in nonextensive statistics [20, 40] (this recipe was not adopted in Ref.[42]). In addition, since all equations must be solved in a self-consistent way, the presence of nonextensive statistical effects influences the many-body interaction mediated by the meson fields.

Especially in regime of low density and high temperature the contribution of the lightest mesons to the thermodynamical potential (and, consequently, to the other thermodynamical quantities) becomes relevant. As quoted before,

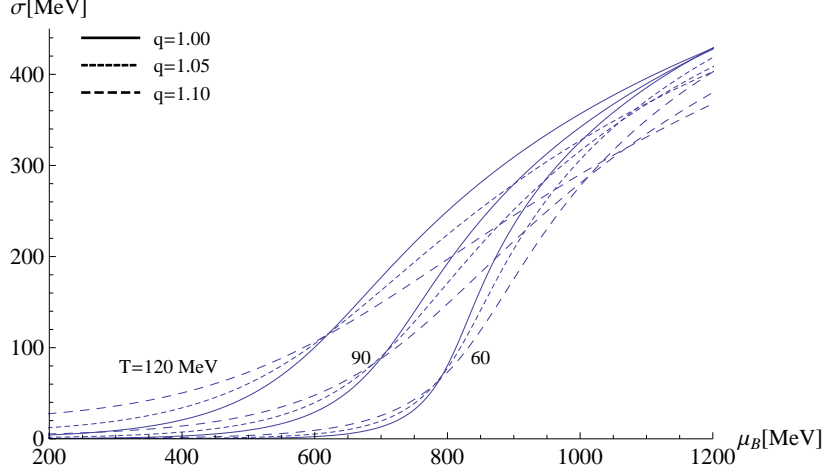


FIG. 1: The  $\sigma$  meson field as a function of baryon chemical potential for different values of temperature (in units of MeV) and  $q$ .

following Ref.s [54, 55], we have included the contribution of pions considering them as a (quasi) ideal gas of nonextensive bosons with effective chemical potentials expressed in terms of the corresponding effective baryon chemical potentials. More explicitly, for  $\pi^+$  mesons we have  $\mu_{\pi^+} = \mu_C \equiv \mu_p - \mu_n$ , where  $\mu_C$  is the electric charge chemical potential. Thus, the corresponding effective pion chemical potential can be written as

$$\mu_{\pi^+}^* \equiv \mu_p^* - \mu_n^* = \mu_p - \mu_n - 2g_\rho \rho, \quad (20)$$

where the last equivalence follows from Eq.(17). Therefore, the  $\rho$  meson field couples to the total isospin density, which receives a contribution from nucleons and pions.

Let us start our numerical investigation by considering the behavior of  $\sigma$ ,  $\omega$  and  $\rho$  meson fields at a fixed value  $Z/A = 0.4$ , for different values of temperature and nonextensive parameter  $q$ . Because meson fields have their source in the baryon and scalar density, which are very sensible to the behavior of the mean occupation number, all meson fields appear to be significantly changed in presence of nonextensive effects.

In Fig. 1, we show the  $\sigma$  meson field as a function of the baryon chemical potential  $\mu_B$ . It is interesting to observe that at lower  $\mu_B$ , in presence of nonextensive effects, the value of the meson field is significantly increased for all values of temperature respect to the standard case, the other way round happens at higher  $\mu_B$ . This important feature is due to the fact that, as already remarked in Section II, for  $q > 1$  and fixed baryon density (or  $\mu_B$ ), the (normalized) mean occupation function is enhanced at high values of its argument and depressed at low values. Being the argument of the mean occupation function  $x_i = \beta(E_i^* - \mu_i^*)$ , in the integration over momentum (energy), at lower  $\mu_B$  (corresponding to lower values of the effective particle chemical potential  $\mu_i^*$ ) the enhanced Tsallis high energy tail weighs much more than at higher  $\mu_B$  where depressed low energy effects prevail and the mean occupation number results to be bigger for the standard Fermi-Dirac statistics. Concerning the antiparticle contribution, the argument of  $\bar{n}_i$  is  $\bar{x}_i = \beta(E_i^* + \mu_i^*)$  and the Tsallis enhancement at high energy tail is favored also at higher  $\mu_B$ . At the same time, higher temperatures (where antiparticle contribution are more relevant) reduce the value of the argument of  $n_i$  and  $\bar{n}_i$ , favoring the extensive distribution. These effects are much more evident for the scalar density  $\rho_S$  (self-consistently related to the  $\sigma$  meson field) where appears  $(n_i)^q$  and particle and antiparticle contributions are summed. The same effect involves also the nucleon effective mass  $M^* = M - g_\sigma \sigma$ , which becomes, respect to the standard case, smaller for lower values of  $\mu_B$  and bigger for higher values, with very relevant consequences for the hadronic EOS [65].

In Fig. 2, we report the  $\omega$  meson field as a function of the baryon chemical potential for different values of temperature and  $q$ . In this case the situation is different from the  $\sigma$  meson, because the  $\omega$  field has its source in the baryon density  $\rho_B$  where appears  $n_i$  and particle and antiparticle contributions are subtracted. At lower temperatures ( $T = 60$  MeV), antiparticle contributions are negligible and we have a behavior similar (although less evident) to the  $\sigma$  field. At higher temperatures ( $T = 120$  MeV), the contributions of antiparticle increase and nonextensive effects vanish at higher  $\mu_B$ .

Finally, in Fig. 3, we report the behavior of the  $\rho$  meson field which depends from the isospin density (let us remember that we have fixed  $Z/A = 0.4$ ). Similar arguments as done for the  $\omega$  meson applies also in this case. The valuable increasing of its absolute value, also for weakly asymmetric nuclear matter, makes  $\rho$  meson very relevant in the hadronic EOS, especially at not too large  $\mu_B$ .

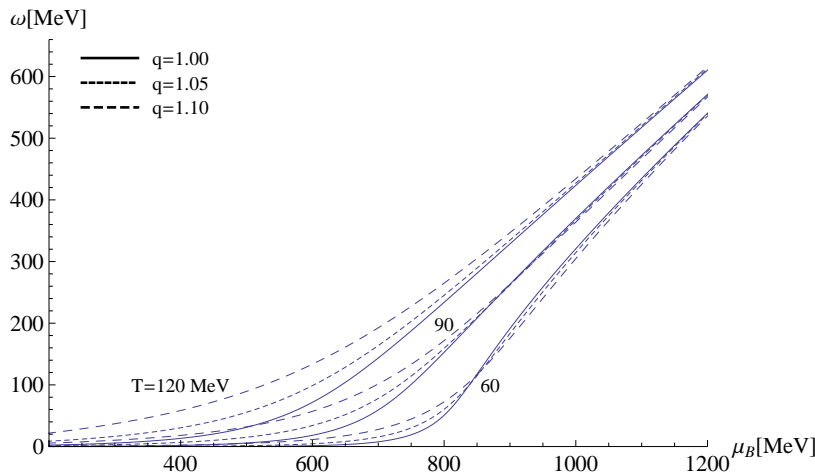


FIG. 2: The  $\omega$  meson field as a function of the baryon chemical potential for different values of temperature and  $q$ .

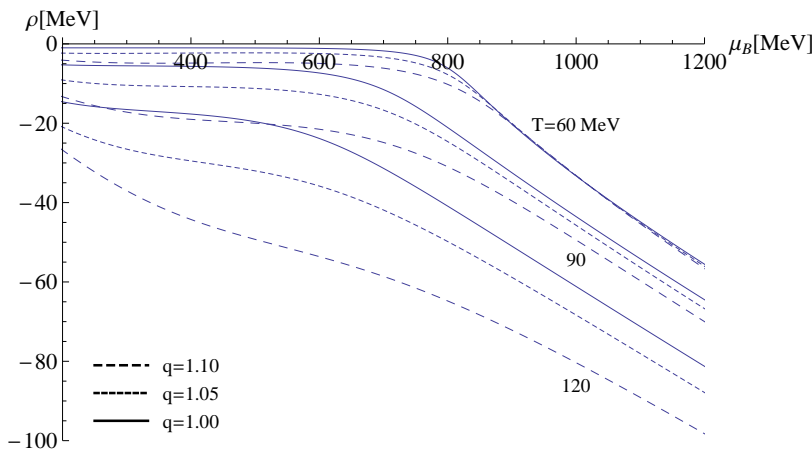


FIG. 3: The  $\rho$  meson field as a function of baryon chemical potential for different values of temperature and  $q$ .

In Fig. 4, the total pressure  $P$  and energy density  $\epsilon$  are plotted as a function of  $\mu_B$  for different values of temperature and  $q$ . The different behavior from  $P$  and  $\epsilon$  reflects essentially the nonlinear combinations of the meson fields and the different functions under integration in Eqs (18) and (19). Concerning the pressure, we have that becomes stiffer by increasing the  $q$  parameter. On the other hand, the behavior of the energy density presents features very similar to the  $\sigma$  field one. At low  $\mu_B$ , nonextensive effects make the energy density greater with respect to the standard case. At medium-high  $\mu_B$ , the standard ( $q = 1$ ) component of the energy density becomes dominant, this effect is essentially due to the reduction of the  $\sigma$  field for  $q > 1$ . The intersection point depends, naturally, on the physical parameters of the system.

#### IV. NONEXTENSIVE QGP EQUATION OF STATE

Concerning the nonextensive quark-gluon EOS, due to its simplicity, we adopt the MIT bag model [56]. In this model, quark matter is described as a gas of free quarks and all non-perturbative effects are simulated by the bag constant  $B$  which represents the pressure of the vacuum.

Following this line, the pressure, energy density and baryon density for a relativistic Fermi gas of quarks in the

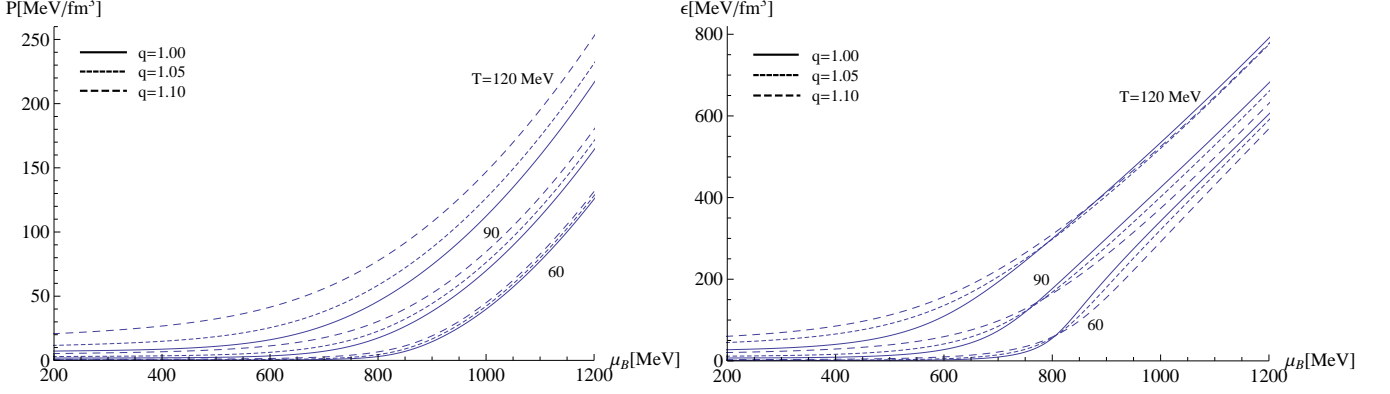


FIG. 4: Pressure (left panel) and energy density (right panel) versus baryon chemical potential for different values of temperature and  $q$ .

framework of nonextensive statistics can be written, respectively, as

$$P_q = \frac{\gamma_f}{3} \sum_{f=u,d} \int_0^\infty \frac{d^3k}{(2\pi)^3} \frac{k^2}{e_f} [n_f^q(k) + \bar{n}_f^q(k)] - B, \quad (21)$$

$$\epsilon_q = \gamma_f \sum_{f=u,d} \int_0^\infty \frac{d^3k}{(2\pi)^3} e_f [n_f^q(k) + \bar{n}_f^q(k)] + B, \quad (22)$$

$$\rho_q = \frac{\gamma_f}{3} \sum_{f=u,d} \int_0^\infty \frac{d^3k}{(2\pi)^3} [n_f(k) - \bar{n}_f(k)], \quad (23)$$

where the quark degeneracy for each flavor is  $\gamma_f = 6$ ,  $e_f = (k^2 + m_f^2)^{1/2}$ ,  $n_f(k)$  and  $\bar{n}_f(k)$  are the  $q$ -deformed particle and antiparticle quark distributions

$$n_f(k) = \frac{1}{[1 + (q-1)(e_f(k) - \mu_f)/T]^{1/(q-1)} + 1}, \quad (24)$$

$$\bar{n}_f(k) = \frac{1}{[1 + (q-1)(e_f(k) + \mu_f)/T]^{1/(q-1)} + 1}. \quad (25)$$

Similar expressions for the pressure and the energy density can be written for gluons treating them as a massless  $q$ -deformed Bose gas with zero chemical potential. Explicitly, we can calculate the nonextensive pressure  $P_g$  and energy density  $\epsilon_g$  for gluons as

$$P_g = \frac{\gamma_g}{3} \int_0^\infty \frac{d^3k}{(2\pi)^3} \frac{k}{[1 + (q-1)k/T]^{q/(q-1)} - 1}, \quad (26)$$

$$\epsilon_g = 3P_g, \quad (27)$$

with the gluon degeneracy factor  $\gamma_g = 16$ . In the limit  $q \rightarrow 1$ , one recovers the usual analytical expression:  $P_g = 8\pi^2/45 T^4$ .

Let us note that, since one has to employ the fermion (boson) nonextensive distributions, the results are not analytical, even in the massless quark approximation. Hence a numerical evaluations of the integrals in Eq.s (21)–(23) and (26) must be performed.

In Fig. 5, we report the total pressure as a function of the baryon chemical potential for massless quarks and gluons, for different values of  $q$  and at fixed value of  $Z/A = 0.4$ . The bag constant is set equal to  $B^{1/4} = 190$  MeV. In presence of nonextensive effects, as in the case of hadronic phase, the pressure is significantly increased even for small deviations from standard statistics.

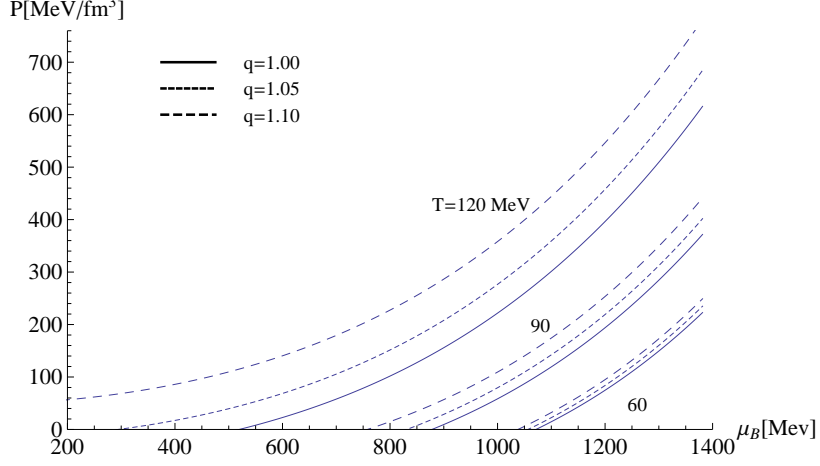


FIG. 5: Pressure of the quark-gluon phase as a function of baryon chemical potential for different values of temperature and  $q$ .

## V. THE HADRON TO QUARK-GLUON PHASE TRANSITION

In this Section we are going to investigate the phase transition from hadronic matter to QGP at finite temperature and baryon chemical potential in the framework of nonextensive statistics.

At this scope, we use the Gibbs formalism applied to systems where more than one conserved charge is present [57]. In fact, because we are going to describe the nuclear EOS, we have to require the global conservation of two "charges": baryon number and electric charge. Each conserved charge has a conjugated chemical potential and the systems is described by two independent chemical potentials:  $\mu_B$  and  $\mu_C$ . The structure of the mixed phase is obtained by imposing the following Gibbs conditions for chemical potentials and pressure

$$\mu_B^{(H)} = \mu_B^{(Q)}, \quad \mu_C^{(H)} = \mu_C^{(Q)}, \quad (28)$$

$$P^H(T, \mu_B, \mu_C) = P^Q(T, \mu_B, \mu_C). \quad (29)$$

Therefore, at a given baryon density  $\rho_B$  and at a given net electric charge density  $\rho_C = Z/A \rho_B$ , the chemical potentials  $\mu_B$  and  $\mu_C$  are univocally determined by the following equations

$$\rho_B = (1 - \chi) \rho_B^H(T, \mu_B, \mu_C) + \chi \rho_B^Q(T, \mu_B, \mu_C), \quad (30)$$

$$\rho_C = (1 - \chi) \rho_C^H(T, \mu_B, \mu_C) + \chi \rho_C^Q(T, \mu_B, \mu_C), \quad (31)$$

where  $\rho_B^{H(Q)}$  and  $\rho_C^{H(Q)}$  are, respectively, the net baryon and electric charge densities in the hadronic (H) and in the quark (Q) phase and  $\chi$  is the fraction volume of quark-gluon matter in the mixed phase. In this way we can find out the phase coexistence region, for example, in the  $(T, \mu_B)$  plane. We are particularly interested in the lower baryon density (baryon chemical potential) border, i.e. the first critical transition density  $\rho_{cr}^I$  ( $\mu_{cr}^I$ ), in order to check the possibility of reaching such conditions in a transient state during a heavy-ion collision at relativistic energies.

In Fig. 6, we report the pressure as a function of baryon chemical potential; as before, we have set  $Z/A = 0.4$  and  $B^{1/4} = 190$  MeV. We can see that at  $T = 60$  MeV, in the range of the considered  $\mu_B$ , the system is in a pure hadronic phase even for the nonextensive index  $q = 1.1$ . At  $T = 90$  and 120 MeV, we have both the first and the second transition critical density for the considered values of  $q$ . In presence of nonextensive effects, the values of the critical densities result to be sensibly reduced with respect to the standard case. This matter of fact is more evident in Fig. 7, where we report the pressure at  $T = 90$  MeV as a function of baryon density (in units of nuclear saturation density  $\rho_0 = 0.153 \text{ fm}^{-3}$ ) (left panel) and energy density (right panel). It is interesting to observe that pressure as a function of baryon density (or energy density) is stiffer in the pure hadronic phase for  $q > 1$  but appears a strong softening in the mixed phase. This feature results in significant changes in the incompressibility and may be particularly important in identifying the presence of nonextensive effects in high energy heavy ion collisions experiments. Related to this aspect, let us observe that possible indirect indications of a significative softening of the EOS at the energies reached at AGS have been discussed several times in the literature [39, 45, 58–60].

In Table I, we report the critical baryon densities and baryon chemical potentials at the beginning (index  $I$ ) and at the end of the mixed phase (index  $II$ ) for different values of temperature and  $q$ .



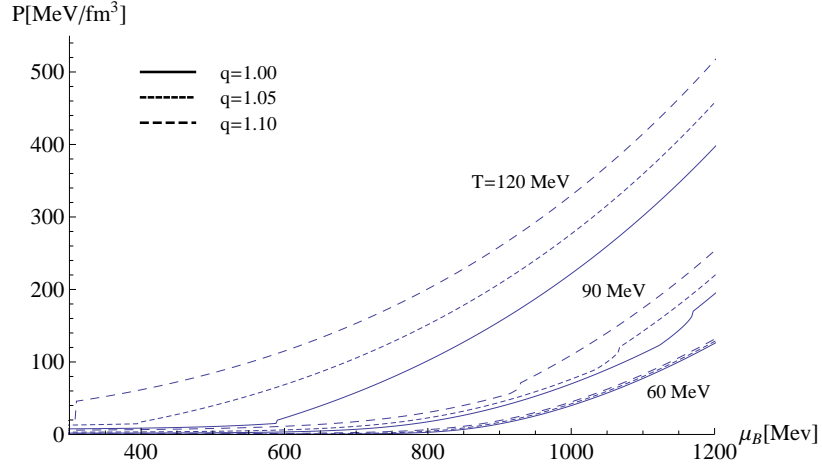


FIG. 6: Pressure versus baryon chemical potential in the mixed phase for different values of temperature and  $q$ .

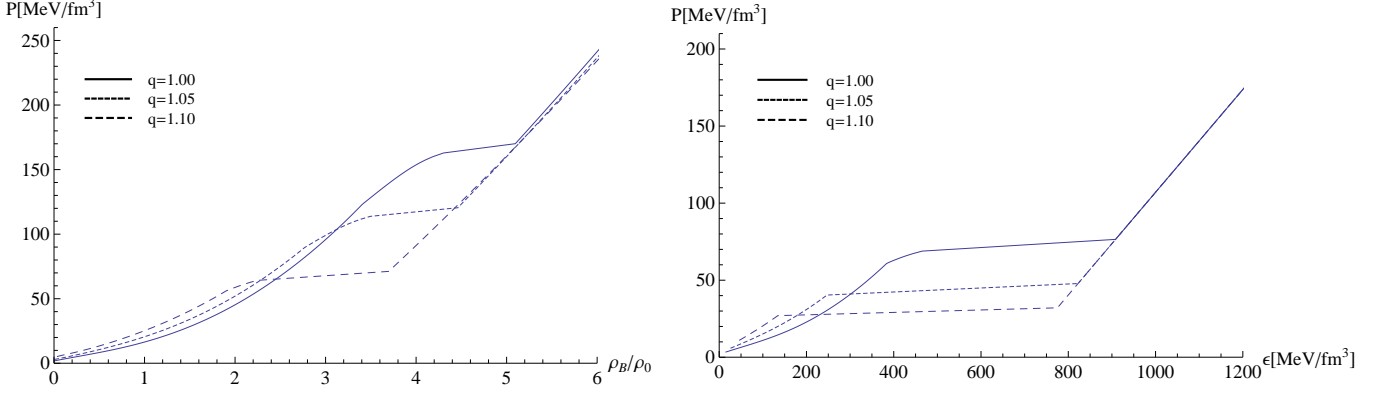


FIG. 7: Pressure as a function of baryon density (left panel) and energy density (right panel) in the mixed phase for different values of  $q$ . The temperature is fixed at  $T = 90$  MeV.

$T = 60$ MeV	$\rho_{\text{cr}}^I/\rho_0$	$\rho_{\text{cr}}^{II}/\rho_0$	$\mu_{\text{cr}}^I$ [MeV]	$\mu_{\text{cr}}^{II}$ [MeV]
$q = 1.00$	5.75	9.10	1503	1569
$q = 1.05$	5.56	8.88	1472	1537
$q = 1.10$	5.33	8.65	1437	1502
$T = 90$ MeV	$\rho_{\text{cr}}^I/\rho_0$	$\rho_{\text{cr}}^{II}/\rho_0$	$\mu_{\text{cr}}^I$ [MeV]	$\mu_{\text{cr}}^{II}$ [MeV]
$q = 1.00$	3.41	5.09	1123	1170
$q = 1.05$	2.77	4.46	1034	1068
$q = 1.10$	1.91	3.69	916	927
$T = 120$ MeV	$\rho_{\text{cr}}^I/\rho_0$	$\rho_{\text{cr}}^{II}/\rho_0$	$\mu_{\text{cr}}^I$ [MeV]	$\mu_{\text{cr}}^{II}$ [MeV]
$q = 1.00$	0.45	1.93	588	616
$q = 1.05$	0.20	1.33	383	396
$q = 1.10$	0.08	0.71	184	201

TABLE I: Critical baryon densities and baryon chemical potentials at the beginning (index  $I$ ) and at the end (index  $II$ ) of the mixed phase for different values of temperature and nonextensive parameter  $q$ .

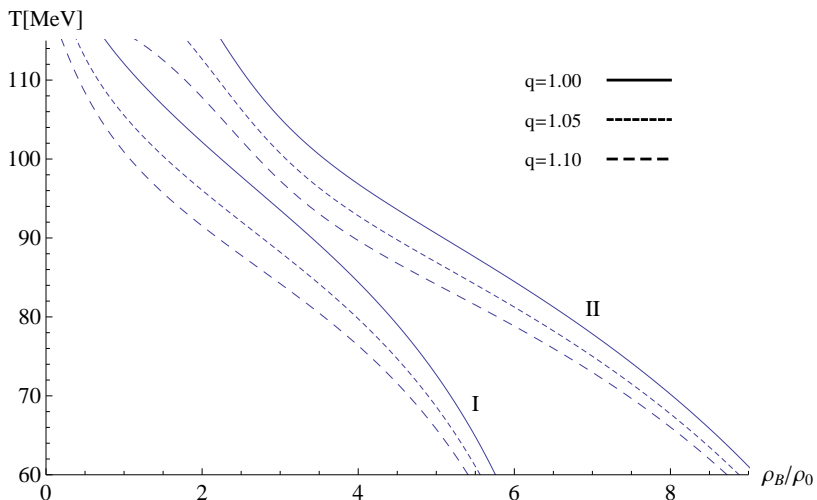


FIG. 8: Phase diagram  $T - \rho_B$  for different values of  $q$ . The curves with index  $I$  and  $II$  indicate, respectively, the beginning and the end of the mixed phase.

In Fig. 8, it is reported the phase diagram in the plane  $T - \rho_B$  for different values of  $q$ . The curves labelled with the index  $I$  and  $II$  represent, respectively, the beginning and the end of the mixed phase. For  $q > 1$ , both the first and the second critical densities are sensibly reduced, even if the shape of the mixed phase is approximately the same. Related to this aspect, let us mention that the simplest version of the MIT bag model, considered in this investigation, appears to be not fully appropriate to describe a large range of temperature and density. To overcome this shortcoming, a phenomenological approach can therefore be based on a density or temperature dependent bag constant [45, 61–63]. Moreover, as discussed in the Introduction, in regime of high temperature and small baryon chemical potential the first order phase transition may end in a (second order) critical endpoint with a smooth crossover. These features cannot be incorporated in the considered mean field approach. In our investigation, because we are focusing to nonextensive statistical effects on the nuclear EOS, instead of introducing additional parameterizations, we work with a fixed bag constant and limit our analysis to a restricted range of temperature and density, region of particular interest for high energy compressed nuclear matter experiments.

Let us now explore in more details the variation of the first transition baryon density  $\rho_{cr}^I$  as a function of different physical parameters. In Fig. 9, we report the dependence of  $\rho_{cr}^I$  as a function of  $Z/A$  for different values of  $q$ . It is interesting to note a significant reduction of  $\rho_{cr}^I$  in presence of nonextensive statistics; this effect is remarkable especially for  $T = 90$  MeV and is essentially a consequence of the  $\rho$  meson field behavior in the hadronic phase.

In Fig. 10 (left panel), we show the first critical baryon density as a function of the bag constant for different values of nonextensive parameter  $q$ . Obviously, by increasing the bag constant we have a corresponding increasing of  $\rho_{cr}^I$ . However, this effect depends on the temperature and nonextensive parameter  $q$ . Finally, in the right panel of Fig. 10, we show the variation of  $\rho_{cr}^I$  as a function of the nonextensive index  $q$  for different values of temperature and  $B^{1/4} = 190$  MeV. At  $T = 60$  MeV, we can see only a little reduction in the first critical density also for large deviations from the standard statistics; on the other hand, the reduction becomes more pronounced at larger temperatures.

## VI. CONCLUSIONS

To summarize, we have studied the main features of the nuclear EOS in the hadronic and quark-gluon phase and the possible formation of a consequent mixed phase in presence of nonextensive statistical effects. We have focused our investigation in regime of finite temperature and baryon chemical potential, reachable in high-energy heavy-ion collisions, for which the deconfinement phase transition can be still considered of the first order. From a phenomenological point of view, the nonextensive index  $q$  is considered here as a free parameter, even if, actually should not be treated as such because, in principle, it should depend on the physical conditions generated in the reaction, on the fluctuation of the temperature and be related to microscopic quantities (such as, for example, the mean interparticle interaction length, the screening length and the collision frequency into the parton plasma). We have restricted our investigation for small deviations from the standard statistics and for values  $q > 1$  because, as quoted in the Introduction, these values were obtained in several phenomenological studies in high energy heavy ion

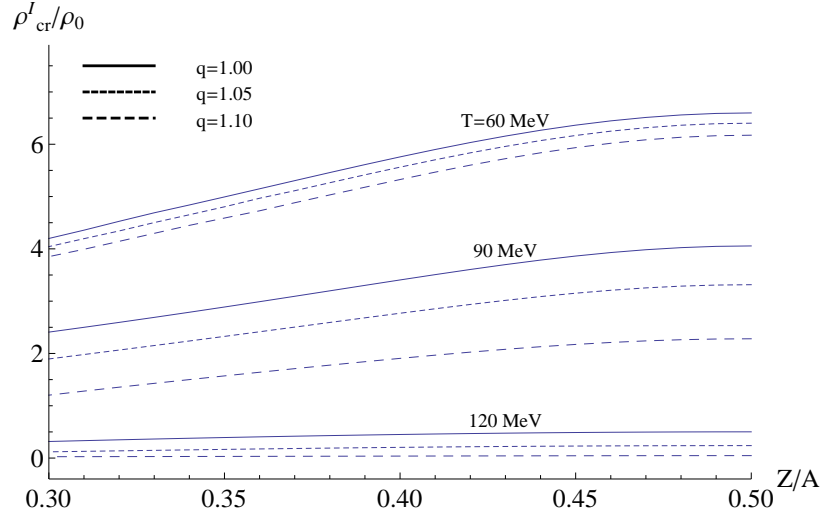


FIG. 9: Variation of the first transition baryon density as a function of the net electric charge fraction  $Z/A$  and for different values of  $q$ .

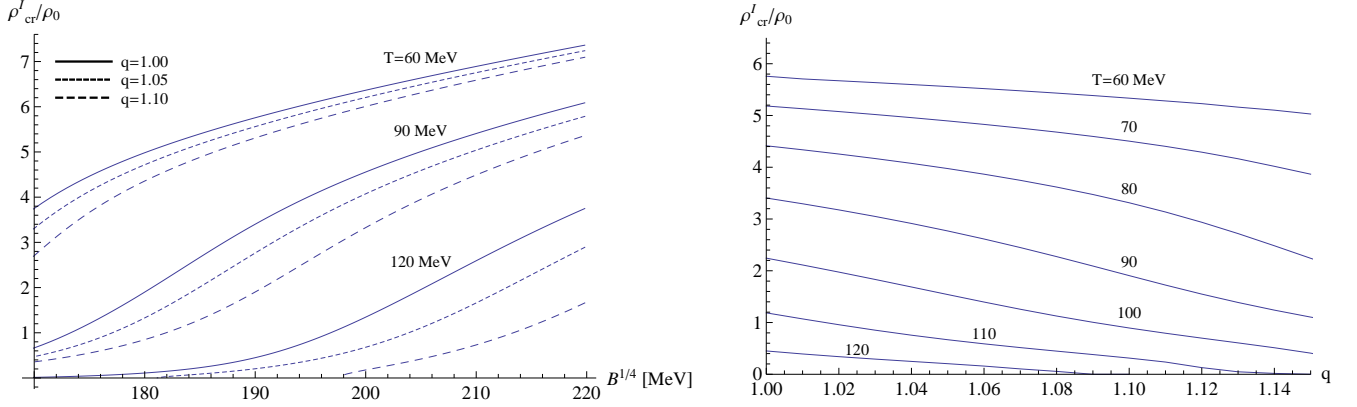


FIG. 10: Variation of the first transition baryon density as a function of the bag constant (left panel) and nonextensive index  $q$  (right panel) for different temperatures.

collisions. In this context, it is relevant to observe that by fitting experimental observable at  $q > 1$ , the temperature (or slope) parameter  $T$  is usually minor of the one obtained in the standard Boltzmann-Gibbs statistics ( $q = 1$ ) [24, 32]. This feature is also present in the considered nuclear equation of state because, at fixed energy per particle  $E/N$ , we obtain for  $q > 1$  lower values of temperature respect to the standard case. Moreover, let us remember that, in the diffusional approximation, a value  $q > 1$  implies the presence of a superdiffusion among the constituent particles (the mean square displacement obeys to a power law behavior  $\langle x^2 \rangle \propto t^\alpha$ , with  $\alpha > 1$ ) [64].

In the first part of the work, we have investigated the hadronic equation of state and the role played by the meson fields in the framework of a relativistic mean field model which contains the basic prescriptions of nonextensive statistical mechanics. We have shown that, also in presence of small deviations from standard Boltzmann-Gibbs statistics, the meson fields and, consequently, the EOS appear to be sensibly modified. In the second part, we have analyzed the QGP proprieties using the MIT Bag model and also in this case the EOS becomes stiffer in presence of nonextensive effects. Finally, we have studied the proprieties of the phase transition from hadronic matter to QGP and the formation of a relative mixed phase by requiring the Gibbs conditions on the global conservation of baryon number and electric charge fraction. We have seen that nonextensive effects play a crucial role in the deconfinement phase transition. Moreover, although pressure as a function of baryon density is stiffer in the hadronic phase, we have shown that a strong softening in the mixed phase takes place in presence of nonextensive statistics. Such a behavior

implies an abruptly variation in the incompressibility and could be considered as a signal of nonextensive statistical effects in high energy heavy ion collisions.

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